Math 115

Spring 2017

Lecture 27

① Simplify: $\dfrac{x+8}{x^2-5x-6} + \dfrac{x+1}{x^2-4x-5}$

$= \dfrac{(x+8)(x-5)}{(x-6)(x+1)(x-5)} + \dfrac{(x+1)(x-6)}{(x-5)(x+1)(x-6)}$

$= \dfrac{x^2-5x+8x-40+x^2-6x+x-6}{(x-6)(x+1)(x-5)} = \boxed{\dfrac{2x^2-2x-46}{(x-6)(x+1)(x-5)}}$

$\dfrac{4}{2x^2+5x-3} - \dfrac{2}{x+3}$

$= \dfrac{4}{(2x-1)(x+3)} - \dfrac{2(2x-1)}{(x+3)(2x-1)} = \quad = \dfrac{4-2(2x-1)}{(2x-1)(x+3)}$

$= \dfrac{4-4x+2}{(2x-1)(x+3)} = \boxed{\dfrac{6-4x}{(2x-1)(x+3)}}$

$$\frac{8}{x^2+6x+5} - \frac{3x}{x^2+4x-5} + \frac{2}{x^2-1}$$

$$= \frac{8(x-1)}{(x+5)(x+1)(x-1)} - \frac{3x(x+1)}{(x+5)(x-1)(x+1)} + \frac{2(x+5)}{(x+1)(x-1)(x+5)}$$

$$= \frac{8x-8-3x^2-3x+2x+10}{(x+5)(x+1)(x-1)} = \boxed{\frac{-3x^2+7x+2}{(x+5)(x+1)(x-1)}}$$

Solve: $3 - \dfrac{6}{x} = x+8$ LCD $= x$

E.V.: 0

$$x \cdot 3 - \cancel{x} \cdot \frac{6}{\cancel{x}} = x \cdot x + x \cdot 8$$

$$3x - 6 = x^2 + 8x \implies x^2 + 8x - 3x + 6 = 0$$

$$x^2 + 5x + 6 = 0$$

$$(x+3)(x+2) = 0$$

$$x+3 = 0 \qquad x+2 = 0$$

$$\boxed{x = -3} \qquad \boxed{x = -2}$$

$$\{-3, -2\}$$

Solve $\dfrac{1}{x-7} = \dfrac{6x}{x^2-5x-14} - \dfrac{3}{x+2}$

LCD = $(x-7)(x+2)$ E.V.: 7 & -2

$(x-7)(x+2) \cdot \dfrac{1}{x-7} = (x-7)(x+2) \cdot \dfrac{6x}{x^2-5x-14} - (x-7)(x+2) \cdot \dfrac{3}{x+2}$

$x+2 = 6x - 3(x-7)$ $x - 3x = 21 - 2$

$x+2 = 6x - 3x + 21$ $-2x = 19$

$x+2 = 3x + 21$ $\boxed{x = \dfrac{-19}{2}}$

$\left\{ \dfrac{-19}{2} \right\}$

Solve: $x + \dfrac{x}{x-5} = \dfrac{5}{x-5} - 7$

LCD = $x-5$ E.V.: 5

$x(x-5) + x = 5 - 7(x-5)$

$x^2 - 5x + x = 5 - 7x + 35$ $x + 8 = 0$

$x^2 - 4x + 7x - 40 = 0$ $\boxed{x = -8}$

$x^2 + 3x - 40 = 0$ $x - 5 = 0$

$(x+8)(x-5) = 0$ $\cancel{x=5}$

$\{ -8 \}$

$$\boxed{\dfrac{2}{x+3}} + \boxed{\dfrac{5-6x}{x^2+2x-3}} = \boxed{\dfrac{2x+3}{x-1}}$$

LCD $= (x+3)(x-1)$ E.V.: $-3 \stackrel{.}{\&} 1$

$$2(x-1) + 5 - 6x = (x+3)(2x+3)$$

$$2x - 2 + 5 - 6x = 2x^2 + 3x + 6x + 9$$

$$-4x + 3 = 2x^2 + 9x + 9$$

$$-4x + 3 - 2x^2 - 9x - 9 = 0$$

$$-2x^2 - 13x - 6 = 0$$

Multiply by -1

$$2x^2 + 13x + 6 = 0$$

$$(2x+1)(x+6) = 0$$

by Z.F.T.

$2x+1=0 \qquad x+6=0$

$\{-\frac{1}{2}, -6\}$ $\boxed{x = \frac{-1}{2}}$ $\boxed{x = -6}$

Solve   LCD $= (x+2)(x-2)$

$$\dfrac{1}{x+2} + \dfrac{1}{x-2} = \dfrac{4}{x^2-4}$$

E.V.: $-2 \stackrel{.}{\&} 2$

$$x - 2 + x + 2 = 4$$

$$2x = 4$$

~~x=2~~
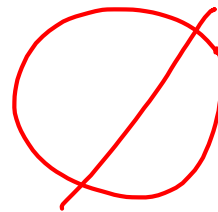
Simplify $\dfrac{\dfrac{5x}{x+2}}{\dfrac{10}{x-2}}$ $LCD = (x+2)(x-2)$

$= \dfrac{5x(x-2)}{\underset{2}{10}(x+2)} = \boxed{\dfrac{x(x-2)}{2(x+2)}}$

Simplify: $\dfrac{\dfrac{x}{y^2} + \dfrac{1}{y}}{\dfrac{y}{x^2} + \dfrac{1}{x}}$

$LCD = x^2 y^2$

$= \dfrac{x^3 + x^2 y}{y^3 + xy^2} = \dfrac{x^2(x+y)}{y^2(y+x)} = \boxed{\dfrac{x^2}{y^2}}$

Simplify: $\dfrac{2 + \dfrac{1}{x}}{4x - \dfrac{1}{x}}$ $LCD = x$

$= \dfrac{2x+1}{4x^2 - 1} = \dfrac{1(2x+1)}{(2x+1)(2x-1)} = \dfrac{1}{2x-1}$

$\dfrac{3x^{-1} + 3y^{-1}}{4x^{-2} - 9y^{-2}} = \dfrac{\dfrac{3}{x} + \dfrac{3}{y}}{\dfrac{4}{x^2} - \dfrac{9}{y^2}} = \boxed{\dfrac{3xy^2 + 3x^2 y}{4y^2 - 9x^2}}$

Hint: $x^{-n} = \dfrac{1}{x^n}$ $LCD = x^2 y^2$

$= \boxed{\dfrac{3xy(y+x)}{(2y+3x)(2y-3x)}}$

The sum of a number and its reciprocal is $\frac{29}{10}$. Find such number.

$\longrightarrow x \qquad \frac{1}{x}$

$$x + \frac{1}{x} = \frac{29}{10}$$

$$10x \cdot x + 10x \cdot \frac{1}{x} = 10x \cdot \frac{29}{10}$$

LCD $= 10x$

$$10x^2 + 10 = 29x$$

$$10x^2 - 25x - 4x + 10 = 0 \qquad 10x^2 - 29x + 10 = 0$$

$$5x(2x-5) - 2(2x-5) = 0 \qquad\qquad P = 100$$

$$\qquad\qquad\qquad\qquad\qquad\qquad S = -29$$

$$(2x-5)(5x-2) = 0 \qquad \frac{5}{2} \ \& \ \frac{2}{5} \qquad 100 \qquad -25 \ \& \ -4$$

$$x = \frac{5}{2} \qquad x = \frac{2}{5}$$

The difference of reciprocals of two consecutive integers is $\frac{1}{6}$. Find such integers.

Two Cons. integers : x , $x+1$

Reciprocal : $\frac{1}{x}$, $\frac{1}{x+1}$

$$\boxed{\frac{1}{x}} - \boxed{\frac{1}{x+1}} = \boxed{\frac{1}{6}}$$

$$6 = x^2 + x$$

$$x^2 + x - 6 = 0$$

LCD $= 6x(x+1)$

$$(x+3)(x-2) = 0$$

$$6(x+1) - 6x = x(x+1)$$

$$\qquad\qquad x = -3 \qquad x = 2$$

$$6x + 6 - 6x = x^2 + x$$

$2 \ \& \ 3 \ \text{or} \ -3 \ \& \ -2$

Jose can do a job in 5 hrs alone.
Mary can do the same job in 4 hrs alone.
How long if they work together?

$$\frac{1}{5} \cdot t + \frac{1}{4} \cdot t = 1 \qquad \frac{t}{5} + \frac{t}{4} = 1$$

$$LCD = 20$$

$$4t + 5t = 20$$

$$9t = 20$$

$$t = \frac{20}{9} \approx 2.2 \text{ hrs}$$

Luna drove 210 miles in the same time that Rowen drove 180 miles.

Rowen was driving 10 mph slower than Luna.

Use table, then find speed for both.

Cat.	r	·	t	= d
Luna	x	·	t	= 210
Rowen	$x-10$	·	t	= 180

$$t_{Luna} = t_{Rowen}$$

$$\frac{\cancel{210}^{7}}{x} = \frac{\cancel{180}^{6}}{x-10}$$

$$\frac{7}{x} = \frac{6}{x-10}$$

$$7(x-10) = 6x$$

Luna @ 70mph $\boxed{x=70}$
Rowen @ 60mph

Adam drove 30 miles in Construction Zone, and 280 miles the rest of way on FWY.

Total trip took 6 hrs.

His speed on FWY was 10 mph faster than 4 times his speed in the Construction Zone. Find Speed on the FWY.

Cat.	r	$\cdot$	t	$= d$
Construction	x	$\cdot$	t_1	$= 30$
FWY	$4x+10$	$\cdot$	t_2	$= 280$

$$\boxed{t_1} + \boxed{t_2} = 6$$

$$\frac{30}{x} + \frac{280}{4x+10} = 6$$

$$\frac{\overset{15}{\cancel{30}}}{x} + \frac{2 \cdot \overset{70}{\cancel{140}}}{\cancel{2}(2x+5)} = \frac{\overset{3}{\cancel{6}}}{1}$$

Solve $\quad \dfrac{15}{x} + \dfrac{70}{2x+5} = 3$

$$LCD = x(2x+5)$$

$$15(2x+5) + 70x = 3x(2x+5)$$

$$30x + 75 + 70x = 6x^2 + 15x$$

$$6x^2 + 15x - 100x - 75 = 0$$

$$6x^2 - 85x - 75 = 0$$

$$(6x + 5)(x - 15) = 0$$

$$\cancel{x = -\tfrac{5}{6}} \qquad \boxed{x = 15} \text{ Construction Zone}$$

$$\boxed{70 \text{ mph on FWY}}$$

John and Jill can do a job together in $\frac{20}{7}$ hrs. working alone, John can do the job 6 hrs longer than Jill working alone. Find time to do work alone.

Jill → x hrs
John → $x+6$ hrs

$$\underbrace{\boxed{\frac{1}{x}}}_{Rate} \cdot \underbrace{\boxed{\frac{20}{7}}}_{Time} + \underbrace{\boxed{\frac{1}{x+6}}}_{Rate} \cdot \underbrace{\boxed{\frac{20}{7}}}_{Time} = 1$$

$$\frac{20}{7x} + \frac{20}{7(x+6)} = 1$$

$$LCD = 7x(x+6)$$

$$20(x+6) + 20x = 7x(x+6)$$

$$20(x+6) + 20x = 7x(x+6)$$

$$20x + 120 + 20x = 7x^2 + 42x$$

$$7x^2 + 42x - 40x - 120 = 0$$

$$7x^2 + 2x - 120 = 0$$

$$(7x + 30)(x - 4) = 0$$

$$\cancel{x = -\frac{30}{7}} \qquad \boxed{x = 4}$$

Jill in 4 hrs
&
John in 10 hrs

Due Monday
WP ch. 13
SG 20